

$$42. \quad \frac{3}{2x} = \frac{1}{4} \quad | \cdot 4x$$

$$6 = x$$

$$\underline{x = 6}$$

$$42 \text{ a.} \quad \frac{5}{3x} = \frac{1}{3} \quad | \cdot 3x$$

$$5 = x$$

$$\underline{x = 5}$$

$$43. \quad \frac{3}{4} - \frac{2}{3x} = \frac{1}{2} + \frac{5}{6x} \quad | \cdot 12x$$

Hinweis: kgV (2, 4, 3x, 6x) = 12x

$$\frac{3}{4} \cdot 12x - \frac{2}{3x} \cdot 12x = \frac{1}{2} \cdot 12x + \frac{5}{6x} \cdot 12x$$

Die Brüche werden gekürzt.

$$9x - 8 = 6x + 10 \quad | - 6x$$

$$3x - 8 = 10 \quad | + 8$$

$$3x = 18 \quad | : 3$$

$$\underline{x = 6}$$

Probe:

$$\text{L: } \frac{3}{4} - \frac{2}{18} = \frac{27 - 4}{36} = \frac{23}{36}$$

$$\text{R: } \frac{1}{2} + \frac{5}{36} = \frac{18 + 5}{36} = \frac{23}{36}$$

$$43 \text{ a.} \quad \frac{8}{x} + \frac{1}{x} - \frac{1}{4} = \frac{6}{2x} + \frac{5}{x} \quad | \cdot 4x$$

$$32 + 4 - x = 12 + 20$$

$$-x + 36 = 32 \quad | - 36$$

$$-x = -4 \quad | \cdot (-1)$$

$$\underline{x = 4}$$

$$44. \quad \frac{x+3}{x-1} = 3 \quad | \cdot (x-1)$$

$$\frac{x+3}{x-1} (x-1) = 3(x-1)$$

Auf der linken Seite wird gekürzt, auf der rechten ausmultipliziert.

$$x + 3 = 3x - 3 \quad | - 3x$$

$$-2x + 3 = -3 \quad | - 3$$

$$-2x = -6 \quad | : (-2)$$

$$\underline{x = 3}$$

$$44 \text{ a.} \quad \frac{4}{x+1} = 1 \quad | \cdot (x+1)$$

$$4 = 1(x+1)$$

$$4 = x + 1 \quad | - 1$$

$$x + 1 = 4 \quad | - 1$$

$$\underline{x = 3}$$

$$45. \quad \frac{3}{x-2} = \frac{6}{x} \quad | \cdot x(x-2)$$

$$3x = 6(x-2)$$

$$3x = 6x - 12 \quad | - 6x$$

$$-3x = -12 \quad | : (-3)$$

$$\underline{x = 4}$$

$$45 \text{ a.} \quad \frac{1}{x-4} = \frac{2}{x} \quad | \cdot x(x-4)$$

$$x = 2(x-4)$$

$$x = 2x - 8 \quad | - 2x$$

$$-x = -8 \quad | \cdot (-1)$$

$$\underline{x = 8}$$

$$46. \quad \frac{5}{x-3} = \frac{10}{x-2} \quad | \cdot (x-3)(x-2)$$

$$5(x-2) = 10(x-3)$$

$$5x - 10 = 10x - 30 \quad | - 10x$$

$$-5x - 10 = -30 \quad | + 10$$

$$-5x = -20 \quad | : (-5)$$

$$\underline{x = 4}$$

$$46 \text{ a. } \frac{3}{x-5} = \frac{12}{x+10} \quad | \cdot (x-5)(x+10)$$

$$3(x+10) = 12(x-5)$$

$$3x + 30 = 12x - 60 \quad | - 12x$$

$$-9x + 30 = -60 \quad | - 30$$

$$-9x = -90 \quad | : (-9)$$

$$\underline{x = 10}$$

$$47. \frac{5}{x+3} - \frac{4}{x-3} = \frac{3}{x-3} \quad | \cdot (x-3)(x+3)$$

Hinweis: $\text{kgV}(x-3, x+3) = (x-3) \cdot (x+3)$

$$\frac{5}{x+3} (x-3)(x+3) - \frac{4}{x-3} (x-3)(x+3) =$$

$$\frac{3}{x-3} (x-3)(x+3)$$

Die Brüche werden gekürzt.

$$5(x-3) - 4(x+3) = 3(x+3)$$

Die Klammern werden ausmultipliziert.

$$5x - 15 - 4x - 12 = 3x + 9$$

$$x - 27 = 3x + 9 \quad | - 3x$$

$$-2x - 27 = 9 \quad | + 27$$

$$-2x = 36 \quad | : (-2)$$

$$\underline{x = -18}$$

$$47 \text{ a. } \frac{3}{x-2} = \frac{2}{x-3} \quad | \cdot (x-2)(x-3)$$

$$3(x-3) = 2(x-2)$$

$$3x - 9 = 2x - 4 \quad | - 2x$$

$$x - 9 = -4 \quad | + 9$$

$$\underline{x = 5}$$

$$48. \frac{1}{x} - \frac{4}{x+1} + \frac{3}{x+2} = 0 \quad | \cdot x(x+1)(x+2)$$

Um sich unnötige Schreibarbeit zu ersparen, wird gleich im Kopf gekürzt. Anstelle von $\frac{1}{x} \cdot x \cdot (x+1)(x+2)$ schreiben wir nur noch $(x+1)(x+2)$ u.s.w.

$$(x+1)(x+2) - 4x(x+2) + 3x(x+1) = 0$$

$$x^2 + x + 2x + 2 - 4x^2 - 8x + 3x^2 + 3x = 0$$

$$-2x + 2 = 0 \quad | - 2$$

$$-2x = -2 \quad | : (-2)$$

$$\underline{x = 1}$$

$$48 \text{ a. } \frac{2}{x+1} - \frac{1}{x} - \frac{1}{x+4} = 0 \quad | x(x+1)(x+4)$$

$$2x(x+4) - (x+1)(x+4) - x(x+1) = 0$$

$$2x^2 + 8x - (x^2 + x + 4x + 4) - x^2 - x = 0$$

$$2x^2 + 8x - x^2 - x - 4x - 4 - x^2 - x = 0$$

$$2x - 4 = 0 \quad | + 4$$

$$2x = 4 \quad | : 2$$

$$\underline{x = 2}$$

$$49. \frac{4x-1}{x} = \frac{15}{4} \quad | \cdot 4x$$

$$4(4x-1) = 15x$$

$$16x - 4 = 15x \quad | - 15x$$

$$x - 4 = 0 \quad | + 4$$

$$\underline{x = 4}$$

$$49 \text{ a. } \frac{3x-4}{2x} = \frac{5}{4} \quad | \cdot 4x$$

$$2(3x-4) = 5x$$

$$6x - 8 = 5x \quad | - 5x$$

$$x - 8 = 0 \quad | + 8$$

$$\underline{x = 8}$$

$$50. \quad \frac{2x-3}{2x-4} + \frac{3x-8}{3x-7} = 2 \quad | \cdot (2x-4)(3x-7)$$

$$(2x-3)(3x-7) + (3x-8)(2x-4) = 2(2x-4)(3x-7)$$

$$6x^2 - 23x + 21 + 6x^2 - 28x + 32 = 12x^2 - 52x + 56$$

$$12x^2 - 51x + 53 = 12x^2 - 52x + 56 \quad | - 12x^2$$

$$-51x + 53 = -52x + 56 \quad | + 52x$$

$$x + 53 = 56 \quad | - 53$$

$$\underline{x = 3}$$

$$50 \text{ a.} \quad \frac{8x+7}{2x+1} - \frac{x+5}{x+2} = 3 \quad | \cdot (2x+1)(x+2)$$

$$(8x+7)(x+2) - (x+5)(2x+1) = 3(2x+1)(x+2)$$

$$8x^2 + 23x + 14 - 2x^2 - 11x - 5 = 6x^2 + 15x + 6$$

$$6x^2 + 12x + 9 = 6x^2 + 15x + 6 \quad | - 6x^2$$

$$12x + 9 = 15x + 6 \quad | - 15x$$

$$-3x + 9 = 6 \quad | - 9$$

$$-3x = -3 \quad | : (-3)$$

$$\underline{x = 1}$$

$$51. \quad \frac{11}{x^2-25} - \frac{x-3}{x-5} = \frac{x-3}{x^2-25} - \frac{x-3}{x+5}$$

Um das kgV der Nenner zu finden, werden diese nach
Möglichkeit zerlegt: $x^2 - 25 = (x + 5)(x - 5)$

$$x - 5 = x - 5$$

$$x + 5 = x + 5$$

$$\text{kgV} = (x - 5)(x + 5)$$

Nun wird die Gleichung mit dem gemeinsamen Nenner
multipliziert. Zugleich wird gekürzt:

$$\frac{11}{x^2-25} \cdot (x+5)(x-5) = 11 \text{ u.s.w.}$$

$$\frac{11}{x^2-25} - \frac{x-3}{x-5} = \frac{x-3}{x^2-25} - \frac{x-3}{x+5} \quad | \cdot (x+5)(x-5)$$

$$11 - (x-3)(x+5) = x-3 - (x-3)(x-5)$$

$$11 - (x^2 - 3x + 5x - 15) = x - 3 - (x^2 - 3x - 5x + 15)$$

$$11 - x^2 + 3x - 5x + 15 = x - 3 - x^2 + 3x + 5x - 15$$

$$-2x + 26 = 9x - 18 \quad | - 9x$$

$$-11x + 26 = -18 \quad | + 11x$$

$$-11x = -44 \quad | : (-11)$$

$$\underline{x = 4}$$

$$51 \text{ a.} \quad \frac{x-1}{2x-1} - \frac{2x-3}{6x-3} + \frac{3x+6}{10x-5} = \frac{5x+1}{8x-4}$$

Hinweis:

$$6x - 3 = 3(2x - 1)$$

$$10x - 5 = 5(2x - 1)$$

$$8x - 4 = 4(2x - 1)$$

Das kleinste gemeinsame Vielfache ist daher
 $60(2x - 1)$

$$\frac{x-1}{2x-1} - \frac{2x-3}{3(2x-1)} + \frac{3x+6}{5(2x-1)} = \frac{5x+1}{4(2x-1)} \quad | \cdot 60(2x-1)$$

$$60(x-1) - 20(2x-3) + 12(3x+6) = 15(5x+1)$$

$$60x - 60 - 40x + 60 + 36x + 72 = 75x + 15$$

$$56x + 72 = 75x + 15 \quad | - 75x$$

$$-19x + 72 = 15 \quad | - 72$$

$$-19x = -57 \quad | : (-19)$$

$$\underline{x = 3}$$

$$52. \quad \frac{3x-1}{3x-6} - \frac{10x+3}{6x^2+12x} = \frac{3x^2+7}{3x^2-12}$$

Um das kgV der Nenner zu finden, werden diese zerlegt:

$$3x - 6 = 3(x - 2)$$

$$6x^2 + 12x = 6x(x + 2)$$

$$3x^2 - 12 = 3(x^2 - 4) = 3(x + 2)(x - 2)$$

$$\text{kgV} = 6x(x - 2)(x + 2)$$

$$\frac{3x-1}{3(x-2)} - \frac{10x+3}{6x(x+2)} = \frac{3x^2+7}{3(x+2)(x-2)} \quad | \cdot 6x(x+2)(x-2)$$

Ein Hinweis zum Ausmultiplizieren und Kürzen:

$$\frac{3x-1}{3(x-2)} \cdot 6x(x-2)(x+2) = (3x-1)2x(x+2)$$

u.s.w.

$$(3x-1)2x(x+2) - (10x+3)(x-2) = (3x^2+7)2x$$

$$(6x^2-2x)(x+2) - (10x^2+3x-20x-6) = 6x^3+14x$$

$$6x^3-2x^2+12x^2-4x-10x^2-3x+20x+6 =$$

$$\begin{array}{rcl}
 6x^3 + 14x & & \\
 6x^3 + 13x + 6 = 6x^3 + 14x & | - 6x^3 & \\
 13x + 6 = 14x & | - 14x & \\
 -x + 6 = 0 & | - 6 & \\
 -x = -6 & | \cdot (-1) & \\
 \underline{x = 6} & &
 \end{array}$$

52 a. $\frac{x-5}{2x+3} - \frac{7}{2x-3} = \frac{2x^2+3}{4x^2-9}$

Hinweis:

$$\begin{array}{l}
 4x^2 - 9 = (2x - 3)(2x + 3) \\
 \text{kgV}(2x + 3, 2x - 3, 4x^2 - 9) = (2x + 3)(2x - 3)
 \end{array}$$

$$\begin{array}{rcl}
 \frac{x-5}{2x+3} - \frac{7}{2x-3} = \frac{2x^2+3}{(2x+3)(2x-3)} & | \cdot (2x+3)(2x-3) & \\
 (x-5)(2x-3) - 7(2x+3) = 2x^2+3 & & \\
 2x^2 - 10x - 3x + 15 - 14x - 21 = 2x^2+3 & & \\
 2x^2 - 27x - 6 = 2x^2+3 & | - 2x^2 & \\
 -27x - 6 = 3 & | + 6 & \\
 -27x = 9 & | : (-27) & \\
 \underline{x = -\frac{1}{3}} & &
 \end{array}$$

53. $\frac{x}{x^2-6x+9} - \frac{5}{x^2-3x} = \frac{1}{x}$

Um das kgV der Nenner zu finden, werden diese zerlegt:

$$\begin{array}{l}
 x^2 - 6x + 9 = (x - 3)^2 = (x - 3)(x - 3) \\
 x^2 - 3x = x(x - 3) \\
 x = x \\
 \text{kgV} = x(x - 3)(x - 3)
 \end{array}$$

$$\begin{array}{rcl}
 \frac{x}{(x-3)^2} - \frac{5}{x(x-3)} = \frac{1}{x} & | \cdot x(x-3)^2 & \\
 x^2 - 5(x-3) = (x-3)^2 & & \\
 x^2 - 5x + 15 = x^2 - 6x + 9 & | - x^2 & \\
 -5x + 15 = -6x + 9 & | + 6x & \\
 x + 15 = 9 & | - 15 & \\
 \underline{x = -6} & &
 \end{array}$$

53 a. $\frac{5}{x-1} - \frac{3x-4}{x^2-2x+1} = \frac{2x+5}{x^2-1}$

$$\begin{array}{l}
 x^2 - 2x + 1 = (x - 1)^2 \\
 x^2 - 1 = (x - 1)(x + 1) \\
 \text{kgV}(x - 1, x^2 - 2x + 1, x^2 - 1) = (x + 1)(x - 1)^2
 \end{array}$$

$$\begin{array}{rcl}
 \frac{5}{x-1} - \frac{3x-4}{(x-1)^2} = \frac{2x+5}{(x-1)(x+1)} & | \cdot (x+1)(x-1)^2 & \\
 5(x+1)(x-1) - (3x-4)(x-1) = (2x+5)(x-1) & & \\
 5x^2 - 5 - (3x^2 - 4x + 3x - 4) = 2x^2 + 5x - 2x - 5 & & \\
 5x^2 - 5 - 3x^2 + 4x - 3x + 4 = 2x^2 + 3x - 5 & & \\
 2x^2 + x - 1 = 2x^2 + 3x - 5 & | - 2x^2 & \\
 x - 1 = 3x - 5 & | - 3x & \\
 -2x - 1 = -5 & | + 1 & \\
 -2x = -4 & | : (-2) & \\
 \underline{x = 2} & &
 \end{array}$$